Lecture 19 CDMA

<u>Ex</u> Digital $\{\pm 1\}$ version of CDMA (uplink)

user 1	wants	to	send	$14 = s_1$	send	$14 \times [1\ 1\ 1\ 1] = \underline{c}_1$
2	"	"		$20 = s_2$	"	$20 \times [1\ 1\ -1\ -1] = \underline{c}_2$
3	"	"		$26 = s_3$		$26 \times [1\ -1\ -1\ 1] = \underline{c}_3$
4	"	"		$-5 = s_4$		$-5 \times [1\ -1\ 1\ -1] = \underline{c}_4$

(+)

Receiver will get $\underline{r} = \quad + \cancel{\text{noise}}$ → 0 (ignored, for now)

$$= \sum_{i=1}^{4} s_i \underline{c}_i$$

(BS) $= [55\ \ 13\ \ -37\ \ 25]$

Q: At the receiver, how can we get the messages back?

A: Use orthogonality

If I want to know s_3,

find $\hat{s}_3 = \frac{1}{4} \langle \underline{r}, \underline{c}_3 \rangle$

Why? Note that

$$\langle \underline{r}, c_3 \rangle = \underline{r} \cdot \underline{c}_3 \quad \text{(dot product)} \quad = \left(\sum_{i=1}^{4} s_i \underline{c}_i \right) \cdot \underline{c}_3$$

$$= \left(\sum_{i=1}^{4} s_i\ \underline{c}_i \cdot \underline{c}_3 \right) = s_3 \underbrace{\underline{c}_3 \cdot \underline{c}_3}_{} = 4 s_3$$

$\underline{c}_i \cdot \underline{c}_j = 0$ for $i \neq j$

$[1\ -1\ -1\ 1] \cdot [1\ -1\ -1\ 1]$
$= 1 + 1 + 1 + 1 = 4$

MATLAB:

Define the code matrix $C = \begin{bmatrix} \boxed{\underline{c}_1} \\ \boxed{\underline{c}_2} \\ \boxed{\underline{c}_3} \\ \boxed{\underline{c}_N} \end{bmatrix}$

A matrix whose rows are orthogonal

Note that $C C^T = N \times I$

Define the message (row) vector

$\underline{s} = [s_1 \quad s_2 \quad s_3 \quad s_4]$ ← combine the messages from all users into one row vector.

$$\underline{r} = s_1 \underline{c}_1 + s_2 \underline{c}_2 + \cdots = \boxed{\underline{S} \times C} + \text{noise} \xrightarrow{} 0 \text{ (for now)}$$

$$[s_1 \quad s_2 \quad s_3 \quad s_4] \begin{bmatrix} \underline{c}_1 \\ \underline{c}_2 \\ \underline{c}_3 \\ \underline{c}_4 \end{bmatrix}$$

For decoding,

$$\hat{s}_j = \frac{1}{N} \langle \underline{r}, \underline{c}_j \rangle = \frac{1}{N} \underline{r}\, \underline{c}_j^T$$

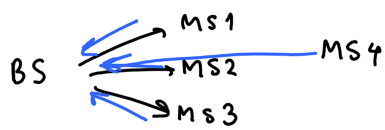
$$\hat{\underline{s}} = [\hat{s}_1 \quad \hat{s}_2 \quad \hat{s}_3 \quad \hat{s}_4]$$

$$= \left[\frac{1}{N} \underline{r}\, \underline{c}_1^T \quad \frac{1}{N} \underline{r}\, \underline{c}_2^T \quad \frac{1}{N} \underline{r}\, \underline{c}_3^T \quad \frac{1}{N} \underline{r}\, \underline{c}_4^T\right]$$

$$= \frac{1}{N} \underline{r} \left[\underline{c}_1^T \quad \underline{c}_2^T \quad \underline{c}_3^T \quad \underline{c}_4^T\right] = \frac{1}{N} \underline{r} \begin{bmatrix} \underline{c}_1 \\ \underline{c}_2 \\ \underline{c}_3 \\ \underline{c}_4 \end{bmatrix}^T = \boxed{\frac{1}{N} \underline{r}\, C^T}$$

$$= \frac{1}{N} (\underline{S} \times C) C^T = \frac{1}{\cancel{N}} \underline{s}\, \cancel{N} I = \underline{s} I = \underline{s}$$

Synchronous CDMA



To Tx, each user only needs his/her own code.

Downlink	Uplink
BS want to send s_i to user i	User i wants to send s_i
BS transmits	User i will transmit $s_i \underline{c}_i$
$\underline{x} = \underline{s} C$ ← Easy to sync.	All $s_i \underline{c}_i$ are combined in the air
	$\underline{x} = \sum_i s_i \underline{c}_i$ ← difficult to sync.
	+ noise
	At receiver (BS)
Each user gets	$\underline{r} = \underline{x} + \text{noise}$
$r = x + \text{noise}$	

$= \underline{s}C + \text{noise}.$

User j recovers its message by

$$\hat{s}_j = \frac{1}{N}\underline{r}\,\underline{c}_j^T$$

Each User only needs his/her own code to decode.

$= \sum_i s_i \underline{c}_i + \text{noise}$

$= \underline{s}C + \text{noise}$

Decode

$$\hat{s} = \frac{1}{N}\underline{r}c^T$$

Summary: CDMA

Transmit: $\underline{s}C$

Decode by $\hat{s} = \frac{1}{N}\underline{r}c^T$ $= \frac{1}{N}(\underline{s}\underbrace{c)c^T}_{NI}$

"c^{-1}" $= \frac{1}{N}c^T$

Simpler C ...

$$C = I = \begin{bmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & 1 & \\ & & & & 1 \end{bmatrix} \Rightarrow \text{TDMA system.}$$

Ex. 3 users

$\underline{c}_1 = [1\ 0\ 0]$

$\underline{c}_2 = [0\ 1\ 0]$

$\underline{c}_3 = [0\ 0\ 1]$

$$\left.\begin{matrix} s_1 = 5 \\ s_2 = 4 \\ s_3 = -1 \end{matrix}\right\} \Rightarrow s_1\underline{c}_1 + s_2\underline{c}_2 + s_3\underline{c}_3 = [5\ \ 4\ \ -1]$$

Lecture 21

Review CDMA: rows of C are orthogonal

$\underline{r} = \underline{s}C + \underline{n}$

$\Downarrow$

$\hat{\underline{s}} = \frac{1}{N}\underline{r}C^H$ $\quad \frac{1}{N}CC^H = I$

$(c)\left(\frac{1}{N}c^H\right) = I$

$\left(\frac{1}{N}c\right)\left(c^H\right) = I$

$\left(\frac{1}{\sqrt{N}}c\right)\left(\frac{1}{\sqrt{N}}c^H\right) = I$

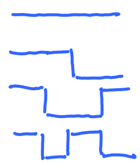
Today: "construct" C [Section 4.7 in Slides]

Today: "construct" C [Section 4.7 in Slides] $\left(\frac{1}{\sqrt{N}}C\right)\left(\frac{1}{\sqrt{N}}C^H\right) = I$

(Last time we've seen that one possible C is the identity matrix. This gives TDMA system.)

So far, we use

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$



$c_1(t)$
$c_2(t)$
$c_3(t)$
$c_4(t)$